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On the origin of Hawking mini black-holes and the cold early Universe

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Summary. Enhancement of inhomogeneities on scales much smaller than the horizon size can easily be achieved if one adopts the most recent equation of state for high-density matter, which predicts the existence of a phase transition. The collective character of the transition might be important in the study of the origin of mini black-holes.

1 Introduction

One of the most interesting suggestions in the field of black holes has recently been made by Hawking (1974, 1975), namely that, besides the 'classical' black holes of solar or super-solar masses, the Universe may be endowed with 'quantum' or mini black-holes of the size of a proton, around which many quantum phenomena take place. Such mini black-holes lose mass by emitting particles, thus raising the temperature, until they eventually get too hot and explode. It is estimated that $M_c = 10^{15}$ g is the mass of the black holes exploding today, while black holes with $M < M_c$ have already exploded. The lifetime of a black hole is proportional to M^3. A simple argument leading to this result is as follows. If a black hole emits like a blackbody, the dominant wavelength λ goes like T^{-1} (we shall use $\hbar = c = 1$). Since $\lambda \sim R_s \sim GM$, then $T \sim G^{-1}M^{-1}$. The luminosity of a black hole of radius R_s is $L \sim T^4 R_s^2 \sim G^{-2}M^{-2}$, so that its lifetime $\tau \sim M/L$ goes like G^2M^3 or, restoring the units,

$$\tau = 10^{10} \left(\frac{M}{10^{15}\,\mathrm{g}}\right) \mathrm{yr}. \tag{1}$$

Since the Universe is about 10^{10} yr old, the critical value $M_c = 10^{15}$ g is particularly important to us. Hawking's theory does not prefer any M, i.e. the mini black-holes which supposedly originated in the early stages of the Big Bang could have had any mass. The problem of the generation of black holes and their cosmological significance has been investigated by Carr (1977), who also summarizes the previous work. The main conclusion of his paper is that a hot universe would tend to produce either too many black holes to be consistent with observation, or too few to be interesting. For this reason, Carr suggests a cold initial universe. However, since a cold universe has some clear disadvantages (e.g. the unlikely production of helium), one should investigate other possibilities before overthrowing the hot universe.

It is the purpose of this paper to show that recent studies indicate that, due to the exchange of attractive spin-2 f^0 mesons among nucleons, (1) the equation of state $p = p(\rho)$ is greatly softened in the high-density regime, and (2) a phase transition may exist where

$$c_s^2 = dp/d\rho \tag{2}$$

becomes zero. If so, we can conclude from the dispersion relation

$$\omega^2 = c_s^2 k^2 - 4\pi G\rho$$

that ω^2 is negative for any k, and no Jeans length exists. More precisely, any length will grow with time, irrespective of its relation to the size of the particle horizon. In particular, very small sizes will grow. But this is precisely what Carr (1977) believes cannot be achieved within a hot universe and for that reason he proposes a cold universe. We believe that this is unnecessary and that the same objective can be achieved with the help of the equation of state to be described below. The second advantage of our model is that the initial fluctuation is enhanced with respect to the case when no phase transition is present. From statistical mechanics (Landau & Lifshitz 1958), we know that a fluctuation δ in the number of particles can be written as

$$\delta^2 = \frac{kT}{V} K, \tag{3}$$

where T and V are the temperature and volume of the system and K is the compressibility

$$K^{-1} = n\, dp/dn, \tag{4}$$

n being the baryonic number density. If the baryons form a perfect gas, then $p = nkT$ and (3) reduces to

$$\delta = l^{-3/2}, \tag{5}$$

where l is a typical size of the system. However, near a phase transition, i.e. in the density region where $c_s^2 = dp/dn \simeq 0$, (3) changes to

$$\delta \sim l^{-1/2} \tag{6}$$

(Landau & Lifshitz 1958). Equation (6) constitutes an enhancement of possibly several order of magnitudes over the usual case (5).

2 The equation of state

In the past few years (Canuto 1974, 1975, 1977), we have learned that nucleons repel each other rather strongly in the density region where neutron stars occur. Neutron-star properties, however, lose their fruitfulness as a discriminating tool when the density reaches 10 to 20 times the nuclear density. Even drastic modifications to the $p = p(\rho)$ curve leave masses, moments of inertia, radii and the like, unaltered.

The behaviour at densities just exceeding neutron density is therefore unknown. It is of very little comfort to know that gauge-field models and Hagedorn-type models propose a $p = \frac{1}{3}\rho$ equation of state. They are applicable to regimes of much higher density and not where neutron stars end. Joining a hard ($p = \rho$) equation of state (valid *at* nuclear densities) to a $p = \frac{1}{3}\rho$ equation of state is not only incorrect but unphysical, since nucleons do not go from strong repulsion to freedom in an abrupt manner. Canuto, Datta & Kalman (1978) have recently shown that the hard equation of state must actually be followed by a softening due to the presence of the next meson (after ω), i.e. the spin-2 f^0 meson, which yields attraction.

A quantitative analysis of a relativistic gas of nucleons interacting through spin-2 f^0 (attractive) mesons, ω-(repulsive) mesons and σ-(attractive) mesons has been performed by solving the equations of motion governing the system, i.e. (Datta 1977; Canuto *et al.* 1978)

$$\begin{aligned}
&\{e_a^\mu \gamma^a (1/i)(D_\mu - ig_v A_\mu) + m_N - g_\sigma \sigma\}\ \psi = 0,\\
&(\partial^2 - m_\sigma^2)\,\sigma = g_\sigma \bar{\psi}\,\psi,\\
&m_v^2 \sqrt{-g}\,A_\mu h^{\mu\nu} + \partial_\mu(\sqrt{-g}\,F_{\lambda\rho}\,h^{\lambda\nu} h^{\rho\mu}) = g_v \sqrt{-g}\,\bar{\psi}\,e_a^\mu \gamma^a \psi,\\
&R_{\mu\nu} + m_f^2(\sqrt{-g}\,g_{\mu\nu} - \eta_{\mu\nu}) = \frac{16\pi f^2}{m_N^2}(t_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu}\,t_{\alpha\beta}\,h^{\alpha\beta}).
\end{aligned} \tag{7}$$

The last equation describes the spin-2 f^0 mesons; $t_{\mu\nu}$ is the energy–momentum tensor corresponding to all the fields except the spin-2 field. The quantities e_a^μ are the vierbein fields, given by

$$e^{\mu a} e^{\nu b}\,\eta_{ab} = h^{\mu\nu} = g_{\mu\nu}^{-1};\quad \eta_{ab} = \mathrm{diag}(-1, +1, +1, +1). \tag{8}$$

The conserved total stress-tensor of the system is given by

$$T_\nu^\mu = \theta_\nu^\mu + \sqrt{-g}\,h^{\mu\beta}\,t_{\beta\nu}, \tag{9}$$

where $\theta_{\nu\mu}$ corresponds to the spin-2 particles. Once T_ν^μ is computed, the equation of state $p = p(\rho)$ is then easily evaluated.

The result is shown in Fig. 1. The effect of the spin-2 meson is that of softening the pressure and making it negative, if one believes that the spin-2 f^0 dominates the high-density regime. The softening leads to the desired result. As we discussed before, this computation shows that there is indeed a density n_* at which

$$\frac{dp}{dn} = 0. \tag{10}$$

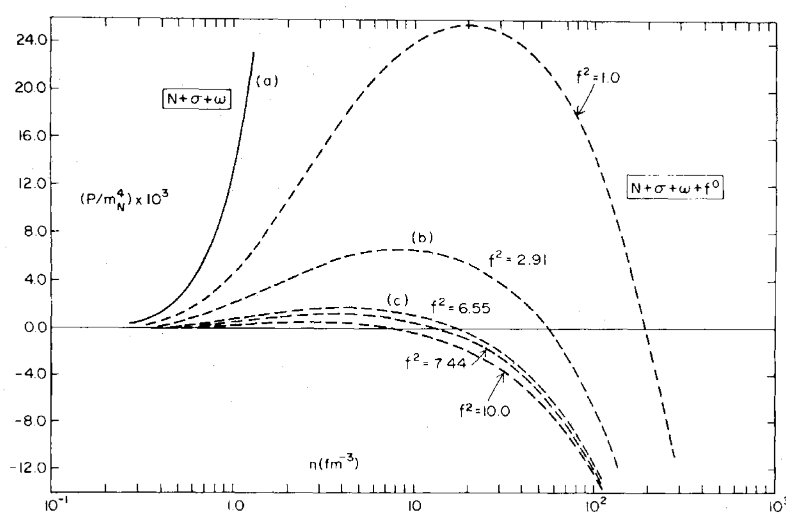

Figure 1. The pressure versus baryonic density n for a system of nucleons (N) interacting via scalar (σ) and vector (ω) particles, case (a). When the spin-2 f^0 meson is introduced, the pressure softens and a phase transition sets in whose location depends on the assumed coupling constant f^2.

Let us now compute the relevant parameters. The phase-transition point depends on the coupling constant for the spin-2 particle. Taking the most reliable coupling constant determined so far, namely

$$f^2 = 2.91, \tag{11}$$

we have

$$n_* \cong 10/\text{fm}^3 = 10^{40}/\text{cm}^3. \tag{12}$$

Since from ordinary cosmology we have

$$n(t) = n_0(R_0/R)^3 = n_0(T/T_0)^3, \quad T = 10^{10}\,t^{-1/2}, \tag{13}$$

and $n_0 = 10^{-6}/\text{cm}^3$, $T_0 = 3\,\text{K}$, the value $n_* = 10^{40}/\text{cm}^3$ corresponds to a time and temperature

$$t_* = 2.4 \times 10^{-12}\,\text{s}, \quad T_* = 6.5 \times 10^{15}\,\text{K}. \tag{14}$$

t_* is late enough for quantum-gravitational effects to be unimportant.

3 Implications

Within the hot model of the early universe, the analysis by Carr (1977) indicates that one can form either too many or too few black holes, depending on the equation of state and the form of the density fluctuations. Assuming a Gaussian distribution for the density fluctuations, the probability that any particular region will evolve into a black hole is

$$P \sim \epsilon \exp(-\alpha^2/2\epsilon), \tag{15}$$

where $p = \alpha\rho$ and ϵ is the amplitude of fluctuations on the horizon scale. A very soft equation of state, $\alpha \ll 1$, will increase P; a hard equation of state, $\alpha = 1$, will produce just the opposite result. At first sight it would seem that the equation of state described in this paper is even softer than say, $p = \frac{1}{3}\rho$, and that therefore our model implies even more black holes than the usual model. This is not so, however, because equation (15) cannot be applied near a phase transition. In fact, the assumption of a Poisson distribution is strictly valid only for small fluctuations. A phase transition is characterized by exactly the opposite situation.

Since we cannot use (15) in our case, we cannot conclude that the present model implies an overproduction of black holes. Even though a full analysis has not yet been done and is not easily done, we believe that some insight can be gained by considering not (15) but the correlation function $\nu(r)$. In general (Landau & Lifshitz 1958)

$$\nu(r) \sim \frac{e^{-\sqrt{a/b}\cdot r}}{r}, \tag{16}$$

where

$$a = \frac{1}{n}\frac{\partial p}{\partial n} = n^{-2}K^{-2}, \quad b = \text{a positive constant}. \tag{17}$$

Away from a critical point, $a \neq 0$ and $\nu(r)$ has a very short range; correlations exist only among nearby regions which act independently. If every region is a potential black hole, many black holes can then be formed.

However, near a phase transition we know that $K \to \infty$ and so $a \to 0$; $\nu(r)$ acquires a longer range: it decreases only like $1/r$ and a correlation of a more collective nature sets in. Fewer regions are independent of one another and therefore a larger fraction of the Universe can be in black holes.

Unfortunately we cannot be more specific and present an analysis as quantitative as the one given by Carr (1975, 1977). The possible existence of a phase transition implies a totally new scenario, one considerably more difficult to analyse. However, since the conclusions are indeed far-reaching (i.e. the possible evidence against a hot Big Bang), we believe that the possibility presented in this paper is interesting enough to deserve a close scrutiny.

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